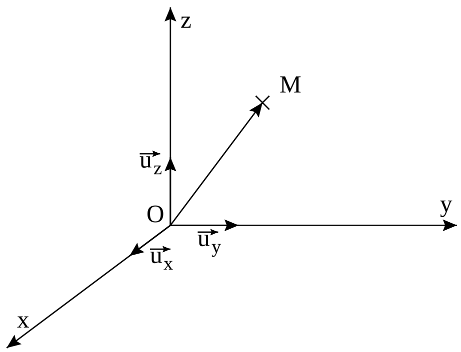
Différents systèmes de coordonnées

Coordonnées cartésiennes

$x,\ y,\ z,\ \vec{u}_x,\ \vec{u}_y,\ \vec{u}_z$

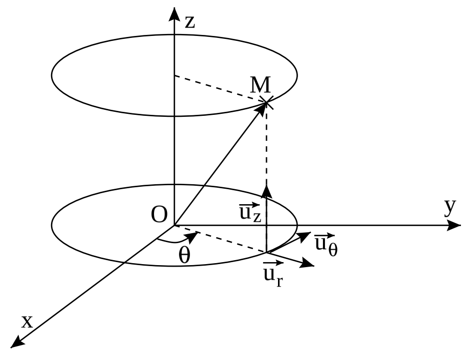


$-\infty < x, y, z < +\infty$

$\overrightarrow{OM} = x\vec{u}_x + y\vec{u}_y + z\vec{u}_z$

Coordonnées cylindriques

$r,\ \theta,\ z,\ \vec{u}_r,\ \vec{u}_\theta,\ \vec{u}_z$

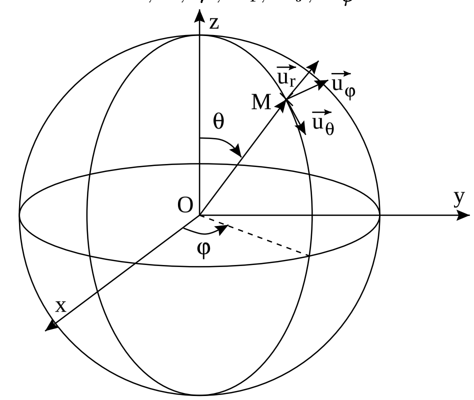


$0 \leqslant r < +\infty,\ 0 \leqslant \theta \leqslant 2\pi,\ -\infty < z < +\infty$

$\overrightarrow{OM} = r\vec{u}_r + z\vec{u}_z$

Coordonnées sphériques

$r,\ \theta,\ \varphi,\ \vec{u}_r,\ \vec{u}_\theta,\ \vec{u}_\varphi$



$0 \leqslant r < +\infty,\ 0 \leqslant \theta \leqslant \pi,\ 0 \leqslant \varphi \leqslant 2\pi$

$\overrightarrow{OM} = r\vec{u}_r$

Relations avec les coordonnées cartésiennes

Cylindriques :

$x = r\cos\theta$

$y = r\sin\theta$

$z = z$

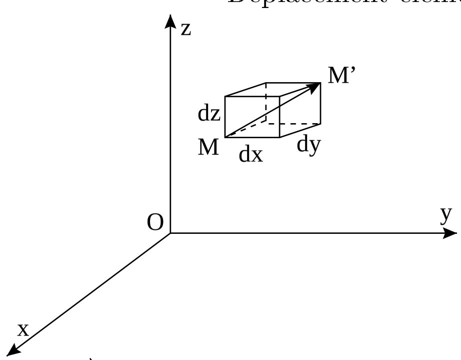
Sphériques :

$x = r\sin\theta\cos\varphi$

$y = r\sin\theta\sin\varphi$

$z = r\cos\theta$

Déplacement élémentaire $\mathrm{d}\vec{l} = \overrightarrow{MM'}$, longueur élémentaire, volume élémentaire, surfaces élémentaires

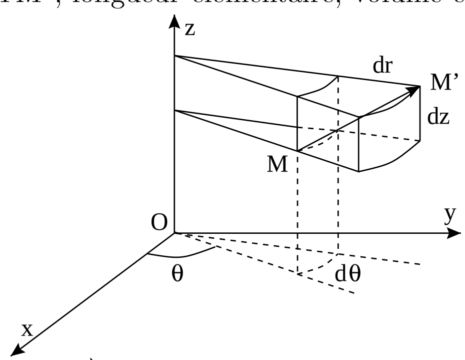


$$\begin{aligned} \mathrm{d}\vec{l} &= \mathrm{d}x\vec{u}_x + \mathrm{d}y\vec{u}_y + \mathrm{d}z\vec{u}_z \\ \mathrm{d}l^2 &= \mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2 \\ \mathrm{d}\tau &= \mathrm{d}x\mathrm{d}y\mathrm{d}z \end{aligned}$$

$x = \text{cst} \ :\ \mathrm{d}S_x = \mathrm{d}y\mathrm{d}z$

$y = \text{cst} \ :\ \mathrm{d}S_y = \mathrm{d}x\mathrm{d}z$

$z = \text{cst} \ :\ \mathrm{d}S_z = \mathrm{d}x\mathrm{d}y$

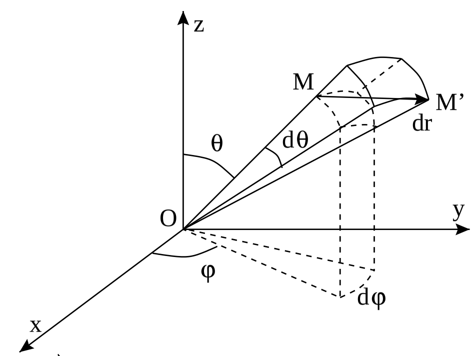


$$\begin{aligned} \mathrm{d}\vec{l} &= \mathrm{d}r\vec{u}_r + r\mathrm{d}\theta\vec{u}_\theta + \mathrm{d}z\vec{u}_z \\ \mathrm{d}l^2 &= \mathrm{d}r^2 + r^2\mathrm{d}\theta^2 + \mathrm{d}z^2 \\ \mathrm{d}\tau &= r\mathrm{d}r\mathrm{d}\theta\mathrm{d}z \end{aligned}$$

$r = \text{cst} \ :\ \mathrm{d}S_r = r\mathrm{d}\theta\mathrm{d}z$

$\theta = \text{cst} \ :\ \mathrm{d}S_\theta = \mathrm{d}r\mathrm{d}z$

$z = \text{cst} \ :\ \mathrm{d}S_z = r\mathrm{d}r\mathrm{d}\theta$



$$\begin{aligned} \mathrm{d}\vec{l} &= \mathrm{d}r\vec{u}_r + r\mathrm{d}\theta\vec{u}_\theta + r\sin\theta\mathrm{d}\varphi\vec{u}_\varphi \\ \mathrm{d}l^2 &= \mathrm{d}r^2 + r^2\mathrm{d}\theta^2 + r^2\sin^2\theta\mathrm{d}\varphi^2 \\ \mathrm{d}\tau &= r^2\sin\theta\mathrm{d}r\mathrm{d}\theta\mathrm{d}\varphi \end{aligned}$$

$r = \text{cst} \ :\ \mathrm{d}S_r = r^2\sin\theta\mathrm{d}\theta\mathrm{d}\varphi$

$\theta = \text{cst} \ :\ \mathrm{d}S_\theta = r\sin\theta\mathrm{d}r\mathrm{d}\varphi$

$\varphi = \text{cst} \ :\ \mathrm{d}S_\varphi = r\mathrm{d}r\mathrm{d}\theta$